

Single particle spectrum of resonant population imbalanced Fermi gases

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We use a T-matrix approximation to calculate the single particle spectrum of the normal state of a gas of Fermionic atoms at low temperature. In the strongly interacting regime of the polarized gas, we find that the spectrum is separated in two branches, leading to a double-peaked radiofrequency spectral feature.

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The experimental observation of fermionic superfluidity in ultracold alkali gases [1] is one of the most important achievements in atomic physics in recent years. By applying a magnetic field [2, 3], experimentalists have been able to control the interaction strength and observed how superfluidity evolves as one increases the attraction between particles from the weak-coupling BCS limit. As was predicted by theory, they saw that the superfluid state of weakly bound pairs continuously evolves to one described as a Bose Einstein condensate (BEC) of tightly bound molecules [4]. In the intermediate region the scattering amplitude saturates the bounds set by the unitarity of the S-matrix, and a strongly interacting superfluid with universal properties appears. The mechanism at the root of fermionic superfluidity is the pairing between atoms in two different states, which we will refer to as \uparrow and \downarrow spin. In recent experiments [5, 6], where the spin relaxation time exceeds all other experimental timescales, researchers have created spin imbalanced Fermi gases, where the ratio of the number of atoms in each spin state $N_{\downarrow}/N_{\uparrow} = x$ is below one. This population imbalance suppresses superfluidity [7, 8], allowing one to study a low temperature normal phase. Unlike the superfluid phase, where symmetries determine most of the systems behavior, the normal phase could be quite exotic. Here we use a T-matrix approximation to study the single particle spectrum of the normal state of an imbalanced Fermi gas, calculating thermodynamic properties and experimental observables.

We will be particularly focused on radio frequency (RF) spectroscopy, where one uses radio waves to drive a transition from one of the two spin states (say \downarrow) to a third, excited state $|ex\rangle$. By exploring the rate of transfer as a function of the frequency of the radio waves, one can gain information about the many-body density matrix. This technique was used to measure the binding energy of pairs in the BEC regime [9], and extended across the resonance to learn about the pairing gap of unitary gas [10]. Recent measurements on imbalanced gases of ^6Li atoms have shown the remarkable feature that the RF spectroscopy is qualitatively unchanged by polarizing the gas [11]. At high temperature (in the normal state of either the equal spin gas or the spin imbalanced gas)

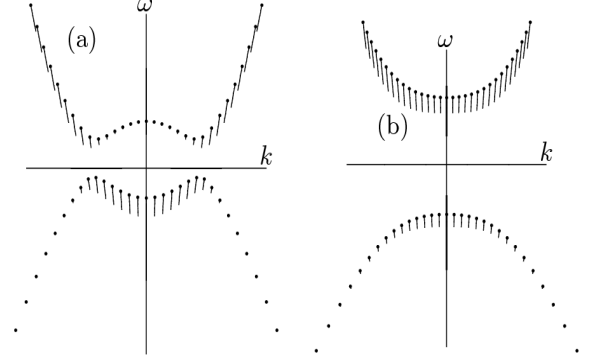


FIG. 1: Three dimensional depiction of mean field spectral density of an unpolarized superfluid gas: (a) “BCS regime”, $\Delta = 0.5\mu$, (b) “BEC regime” $\Delta = -10\mu$. The points are raised a distance above the k - ω plane proportional to the height of the delta-function peaks. These large values of Δ are chosen to emphasize the important qualitative features.

one sees a single peak in the RF spectrum. As the temperature is lowered a second peak appears. In both the unpolarized and spin imbalanced cases the appearance of this second peak occurs at a temperature of order the superfluid transition temperature of the gas with equal spin populations. At lower temperatures yet, the original peak disappears. These results are surprising. Although the unpolarized gas undergoes a superfluid phase transition, the spin imbalanced gas remains in the normal state. How can the normal state of the imbalanced gas have an RF spectrum which is qualitatively indistinguishable from that of the superfluid state of the equal spin gas? Is the ground state of the spin imbalanced gas an exotic state containing noncondensed bosonic pairs?

As we show below, the RF spectra observed by Schunck et al. [11] are consistent with a traditional Fermi liquid scenario, where the low energy excitations can be placed in one-to-one correspondence with those of an ideal Fermi gas. The higher energy excitations of this system are however quite unusual. We find an anomaly in the down-spin spectral density at energies/momenta around the up-spin Fermi energy/momentum. This spectral feature, characterized by a sudden bending of the spectrum, ef-

fectively separates the single particle spectrum into two branches, yielding the two peaks seen in experiments.

Before discussing a theory of the spin imbalanced normal state it is helpful to review the physics of RF spectroscopy in mean-field theory of the superfluid state. For simplicity we will neglect the interaction between the atoms in the $|\text{ex}\rangle$ state and the other two spin states. This approximation should not be quantitatively correct for the recent experiments, where there are several Feshbach resonances in close proximity [12]. Despite the importance of understanding these final state effects [13], one does not expect that they play any qualitative role in the observations. Under these assumptions, particles in the excited internal state have a free dispersion $\epsilon_{\text{ex}}(k) = \epsilon_{\text{ex}}(0) + k^2/2m$, and Fermi's golden rule tells us that the number of atoms transferred by an RF spectroscopy experiment is proportional to [14, 15, 16]

$$I(\nu) = \int \frac{d^3k}{(2\pi)^3} A_{\downarrow}(k, k^2/2m + \mu_{\downarrow} - \nu) n_{\downarrow}(k^2/2m + \mu_{\downarrow} + \nu), \quad (1)$$

where ν is the detuning of the radio frequency field from the free-space splitting between the $|\text{ex}\rangle$ state and the \downarrow state, $n(\epsilon) = (e^{\beta\epsilon} + 1)^{-1}$ is the Fermi function, β is the inverse temperature, and the spectral density $A_{\downarrow}(k, \omega)$ represents how many down-spin single particle states exist at a given momentum and energy (measured from the down-spin chemical potential). Figure 1 shows the mean-field result $A(k, \omega) = v_k^2 \delta(\omega - E_k)$, where the coherence factor is $v_k^2 = (\epsilon_k - E_k)/(2E_k)$, the noninteracting spectrum is $\epsilon_k = k^2/(2m) - \mu$ and the quasiparticle spectrum is $E_k^2 = \epsilon_k^2 + \Delta^2$, with the superfluid gap given by Δ . The spectral density is qualitatively different in the BCS regime ($\mu > 0$) and the BEC one ($\mu < 0$). Qualitatively, the unitary gas is similar to the BCS limit as it has $\mu > 0$. It is important to note that aside from near the places where the two branches become close, most of the spectral weight lies near the free particle spectrum $\omega = \epsilon_k$.

These mean-field spectra can be understood by noting that within the mean-field picture there are two ways to add a down-spin particle with momentum k . One can directly add it (costing energy $\epsilon_{k\downarrow}$) or one can remove an up-spin particle of momentum $q - k$ and simultaneously add a pair with momentum q . In this latter case the change in energy is the difference between the energy of the pair measured from the pair chemical potential $E_b + q^2/2m_b - \mu_b$, and the energy of the up-down particle which was removed. In the superfluid state there is a condensate of $q = 0$ pairs, requiring $E_b - \mu_b = 0$. One therefore finds that the energy to add the spin-down particle in this manner costs energy $-\epsilon_{k\uparrow}$. An avoided crossing between these two modes gives the BCS spectrum. In the BEC limit there is no avoided crossing, and one just has two parabolic bands.

In contrast to the mean field treatment of the super-

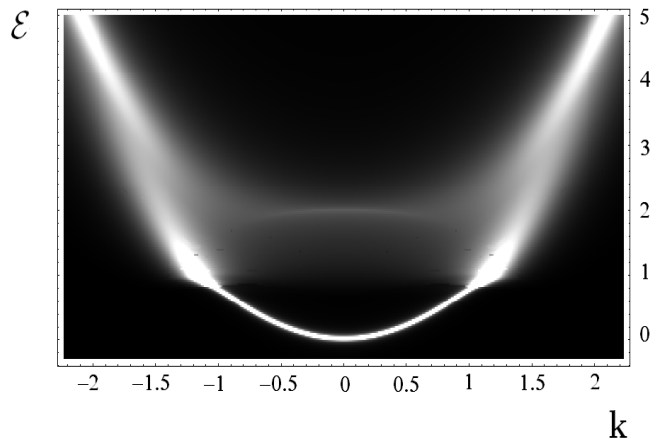


FIG. 2: Spectral density of spin down quasiparticles $A(\epsilon, k)$ at zero temperature. The ϵ axis is normalized by μ_{\uparrow} , the k axis by $\sqrt{2m\mu_{\uparrow}}$, m being the bare mass of atoms. The value of the chemical potential ratio in this plot is $\mu_{\downarrow}/\mu_{\uparrow} = -0.6$, the scattering length is taken to be infinite.

fluid, when one includes fluctuations in the normal state (either above T_c or in the normal state of a spin imbalanced gas), there are many possible channels for adding a single down-spin particle. In principle, one should consider all possible ways of exciting the many body state while adding the particle. Below we will take into account processes where a single spin-up particle hole pair is created. Including all such processes leads to anomalous features in the spectral density of spin-down particles. The spectral density is a nonanalytic function of energy and momenta at the values where the phase space for the creation of particle-hole pairs hits the boundary of the particle-hole continuum. This feature, which is a consequence of the sharp Fermi Surface, corresponds to Kohn anomalies which are observed in the spectra of phonons [17]. At momenta of the order of the Fermi momentum of spin up particles, the spectrum of spin down particles bends and becomes damped, creating a dip in the density of states. This effectively translates into a separation of the spectrum into a low momentum (shifted down from the free spectrum) and a high momentum branch (shifted up from the free spectrum), which gives to the density of states in the normal state (Fig. 2) a structure similar to that in the superfluid state (Fig. 1).

Geometrically, the RF spectroscopy intensity $I(\nu)$ can be found by overlaying the parabola $k^2/2m - \nu + \mu_{\downarrow}$ on the down-spin spectral density graph and quantifying their overlap. Note that since momentum is conserved under absorption of an RF photon, this experiment is very different from a point-contact tunneling experiment in solid state physics. Rather it is equivalent to a solid state photoemission experiment (if one does not momentum-resolve the final state) or a tunneling experiment in which momentum is conserved (such as occurs in tunneling between parallel wires [18]).

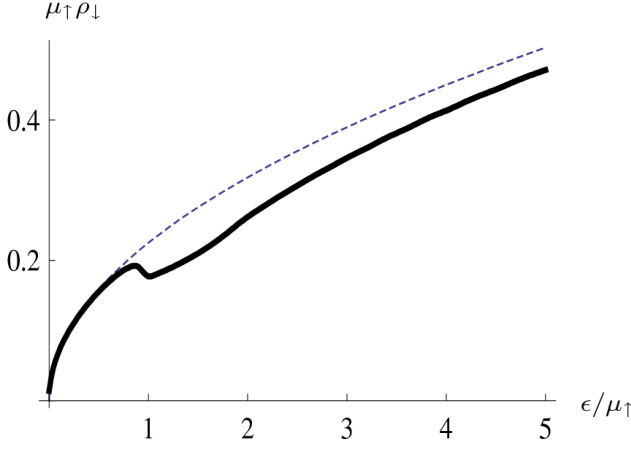


FIG. 3: Single particle density of states $\rho_\downarrow(\epsilon)$, normalized by μ_\uparrow , plotted in the absence (dashed) and in the presence (solid) of interaction as a function of ϵ/μ_\uparrow . The density of states is reduced relative to the noninteracting value near $\epsilon \sim \mu_\uparrow$.

In the superfluid state at finite temperature one sees a bimodal RF spectrum as each of the two branches of the single particle spectrum contributes to the RF line-shape at different frequencies. Since the upper branch asymptotically approaches that of a free gas, one of the two spectral lines begins at $\nu = 0$. The separation δ between the two lines is given by the difference between the $k = 0$ energy of the free parabola and the $k = 0$ energy of the lower branch: $\delta = \sqrt{\mu_\downarrow^2 + \Delta^2} - \mu_\downarrow$. (It is particularly important to realize that this splitting is not simply the gap Δ [19].) At high temperature the contribution from the upper branch dominates, while at low temperature the contribution from the lower branch dominates. On the BCS side of resonance, where pairing mostly occurs near the Fermi surface, both peaks are dominated by unpaired states. As is clear from the spectral density shown in Figure 1(a), the presence of pairs only plays a role very near to the gap. On the other hand, on the BEC side of resonance, the entire lower branch is due to pairs. Clearly, the existence of a shifted low temperature RF peak is not sufficient to conclude that the system is superfluid, a counterexample being a gas of fermions with interaction treated in the Hartree Fock approximation.

To extend this picture to the spin imbalanced gas we must calculate the spectral density $A(k, \omega)$. Since there exists to our knowledge no controlled method for calculating the properties of a Fermi gas at unitarity, we work with the simplest approximation which captures the basic physics – namely the many-body T-matrix approximation which generalizes Noziere and Schmidt-Rinks theory of the BCS-BEC crossover to the spin imbalanced gas. Variants of this approximation have been used by several groups [20, 21]. We use the one described by Combescot et al [22, 23], who have shown that this approxima-

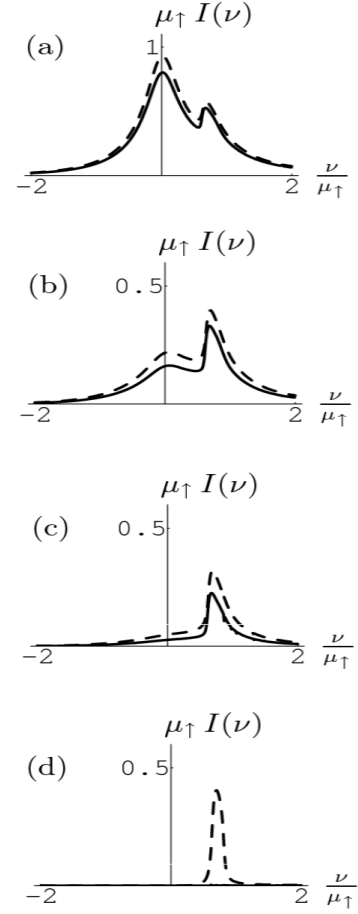


FIG. 4: Calculated RF spectrum in the normal state at unitarity: (a) $T = 4\mu_\uparrow$, (b) $T = 2\mu_\uparrow$, (c) $T = \mu_\uparrow$ (d) $T = 0$. Solid lines correspond to $\mu_\downarrow/\mu_\uparrow = -0.60671$, resulting in vanishing minority species density at $T = 0$, Dashed lines correspond to $\mu_\downarrow/\mu_\uparrow = 0.1$.

tion gives remarkably good agreement with Monte-Carlo calculations in the limit of vanishing downspin density. Specifically we take

$$A_\downarrow(\omega, k) = \text{Im} \left(\omega - \epsilon_k^\downarrow - \Sigma_\downarrow(\omega, k) \right)^{-1} \quad (2)$$

$$\Sigma_\downarrow(\omega, k) = \int dz \Gamma_\downarrow(z, k) / (2\pi(\omega - z)) \quad (3)$$

$$\Gamma_\downarrow(\omega, k) = \int_{0 < \epsilon_k < \omega} d^3q / (2\pi)^3 \Lambda(\omega + \epsilon_q^\uparrow, k + q) \quad (4)$$

$$\Lambda(\omega, k) = 2\text{Im}T(\omega, k) \quad (5)$$

$$T(\omega, k) = (4\pi\hbar^2/m)/(a^{-1} + \Theta(\omega, k)) \quad (6)$$

$$\Theta(\omega, k) = \int \frac{dz}{2\pi(\omega - z)} \int \frac{d^3q}{(2\pi)^3} \left[\frac{1 - f_{k/2+q}^\uparrow - f_{k/2-q}^\downarrow}{\omega - \epsilon_{k/2+q}^\uparrow - \epsilon_{k/2-q}^\downarrow} - \frac{m}{k^2} \right] \quad (7)$$

where $\epsilon_k^\sigma = k^2/2m - \mu_\sigma$, and $f_k^\sigma = \theta(-\epsilon_k^\sigma)$. Note that spectra used to calculate the self-energy are free spec-

tra. Self-consistency should not qualitatively change the spectral density, and may actually make the theory less accurate [24].

The resulting spectral density is shown in Fig. 2, where the separation between a small momentum branch, underdamped and parabolic, and a branch at higher momenta, shifted upward, is clearly recognizable. This two branch structure is similar to the mean-field spectral densities in Fig. 1, except for the fact that the branches are joined by an overdamped continuum. In the limit of vanishing N_{\downarrow} , (occurring at $\mu_{\downarrow} = -0.6067$) we find that the lower branch is described by $E(k) = k^2/2m^*$, with an effective mass $m^* = 1.16$. At sufficiently large momenta, the damping of quasiparticles is vanishingly small. This is because the fundamental interaction is short-ranged, and unitarity of the S-matrix requires that scattering becomes weak at large momenta. Figure 3 shows the integrated density of states

$$\rho(\epsilon) = \frac{1}{2\pi^2} \int_0^\infty dp p^2 A(p, \epsilon) \quad (8)$$

where a dip is clearly visible at energies close to the Fermi level. Here, we mention that a similar dip is found in the spectral density of cuprate superconductors [20].

In Figure 4, we use Eq. 1 to calculate the RF-spectroscopy lineshape. Note that since we are using the zero-temperature spectral density the finite temperature line-shapes are at most qualitative. The general structure, however is generic. At the lowest temperatures only the bottom branch of the spectrum is occupied, and one sees only a single peak, shifted from $\delta = 0$ by an amount proportional to μ_{\uparrow} . In the limit of vanishing downspin density this shift is directly equal to the downspin chemical potential $\mu_{\downarrow} = \Sigma(k=0, \omega=0)$, and provides a model independent way to determine this quantity. As temperature rises the upper branch becomes occupied, resulting in a second peak at lower frequencies. The weight in the second peak grows with the temperature, eventually dwarfing the low temperature peak.

We believe that in a finite temperature calculation of $A(k, \omega)$ one would find that the separation between the two branches would become smaller as temperature increased. This would manifest itself in the low temperature peak slowly moving towards lower energy, merging with the high temperature peak. The splitting should vanish at a characteristic temperature $T^* \sim \mu_{\uparrow}/k_B$.

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